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Transient process of thermally stratifying an initially homogeneous fluid in an enclosure

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1. INTRODUCTION

CONSIDER a homogeneous incompressible Boussinesq fluid in a closed cylinder (radius a, height h) initially at rest. The central axis of the cylinder is parallel to gravity g. At time $t = 0$, a stable, uniform, vertical temperature gradient $\Delta T/h$, where ΔT is the temperature difference between the top and bottom endwalls, is applied to the boundaries and maintained so thereafter. The problem is to find the transient flow and temperature fields developed in the fluid in response to this externally-imposed thermal stratification on the boundaries. The Brunt-Väisälä frequency of the fluid changes from the initial value $N_i = 0$ to the final state value $N_f = (\alpha g\Delta T/h)^{1/2}$, α being the coefficient of volumetric expansion. Situations for which the overall Rayleigh number, $Ra = \alpha g\Delta T h^3/\nu\kappa$, is large and the Prandtl number, $Pr = \nu/\kappa$, is of $O(1)$ will be considered. Here ν is the kinematic viscosity and κ the thermal diffusivity.

The problem posed above is a special case of 'heat-up', a term used by Veronis [1] to describe the general temperature adjustment process of a fluid to the changes in boundary temperatures. Sakurai and Matsuda [2] studied transient motion of an initially stratified ($N_i \neq 0$) Boussinesq fluid in a cylinder in response to a small change in the stratification $\Delta N [\equiv N_f - N_i]$ on the boundaries. Invoking the assumption $\varepsilon \equiv \Delta N/N_f \ll 1$, they obtained analytical solutions for linearized heat-up flows in simple geometries [2, 3]. They clearly demonstrated that a meridional circulation, driven by a buoyant sidewall boundary layer pumping mechanism, redistributes the fluid until a new equilibrium state of stratification N_f is achieved in the fluid. The meridional circulation causes the fluid particles in the inviscid interior to transport its initial temperature (more precisely, its static enthalpy) to a new equilibrium position, at which this temperature equals the corresponding temperature at the container wall [3]. The overall adjustment process is characterized by the buoyant convective time scale, $O(Ra^{1/4}\ N_f^{-1})$, rather than the diffusive time scale, $O(Ra^{1/2}\ N_f^{-1})$.

The present problem of heat-up from an initial state of non-stratification ($N_i = 0$) entails some qualitatively different features from those of linear heat-up described above. The mechanism of meridional circulation induced by buoyant boundary layers is still the most important process. However, since the temperature distribution is initially uniform, the meridional circulation cannot alter the temperature in the interior initially. In this case, the main body of fluid has to be flushed to the boundary layer first; and, while travelling inside the boundary layer, changes in fluid temperature take place. The temperature at a given point in the interior does not change until the arrival of the fluid that has traveled through the boundary layer and returned to the interior. This suggests that a temperature front propagates from the boundaries. Ahead of this front, the fluid remains non-stratified, retaining the uniform temperature of the initial state. Behind the front, the fluid is stratified.

In this note details of the flow and temperature structures of this heat-up process are examined. Comprehensive and accurate flow data have been obtained by employing a finite-difference numerical model. Only the representative and physically illuminating results are presented.

2. FORMULATION

The governing time-dependent Navier–Stokes equations are

$$\frac{\partial u}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(ru^2) - \frac{\partial}{\partial z}(uw) - \frac{1}{\rho_r}\frac{\partial p}{\partial r} + \nu\left(\nabla^2 u - \frac{u}{r^2}\right)$$

$$\frac{\partial w}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(ruw) - \frac{\partial}{\partial z}(w^2) - \frac{1}{\rho_r}\frac{\partial p}{\partial z} + \alpha g(T - T_r) + \nu\nabla^2 w$$

$$\frac{\partial T}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(ruT) - \frac{\partial}{\partial z}(wT) + \kappa\nabla^2 T$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0$$

where

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2};$$

(u, w) are the velocity components in the radial (r) and vertical (z) directions, respectively; t the time; p the reduced pressure; and T the temperature. The equation of state is $\rho = \rho_r[1 - \alpha(T - T_r)]$, in which ρ is the density and subscript r denotes the reference values at the cylinder mid-depth, $z/h = 0.5$.

The associated initial and boundary conditions are

$$u = w = 0,\quad T = T_i \quad \text{at} \quad t = 0$$

$$u = w = 0,\quad T = T_f(z) = T_i + \Delta T(z/h - 0.5) \quad \text{at} \quad r = a$$

$$u = w = 0,\quad T = T_i - 0.5\,\Delta T \quad \text{at} \quad z = 0$$

$$u = w = 0,\quad T = T_i + 0.5\,\Delta T \quad \text{at} \quad z = h.$$

In order to satisfy numerical stability requirements, a thin solid cylinder of very small but finite radius ($r = r_i$) was inserted along the central axis [4]. Thus, $u = \partial w/\partial r = \partial T/\partial r = 0$ at $r = r_i$.

The finite-difference schemes of Warn–Varnas *et al.* [4] on a staggered mesh were chosen for the present problem. The reader is referred to ref. [4] for details on the numerical methods.

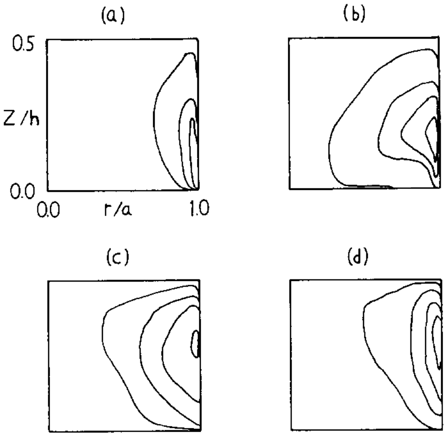

FIG. 1. Contour maps of stream function, ψ, expressed in units of $Ra^{-1/4}h^2N_f$, for case N3 ($Ra = 2.89 \times 10^8$). Times are shown by $\tau = t/t_h$, and $\Delta\psi$ denotes the contour increment: (a) $\tau = 0.014, \psi_{min} = -1.78, \Delta\psi = 0.42$; (b) $\tau = 0.22, \psi_{min} = -2.74, \Delta\psi = 0.70$; (c) $\tau = 0.81, \psi_{min} = -1.01, \Delta\psi = 0.28$; (d) $\tau = 1.20, \psi_{min} = -0.56, \Delta\psi = 0.14$.

3. RESULTS AND DISCUSSION

Computations were performed for the following sets of parameters:

Case N1,

$Ra = 1.45 \times 10^6, \quad N_f = 0.53\ s^{-1}, \quad t_h = 46.4\ s;$

Case N2,

$Ra = 2.89 \times 10^7, \quad N_f = 0.75\ s^{-1}, \quad t_h = 69.5\ s;$

Case N3,

$Ra = 2.89 \times 10^8, \quad N_f = 7.48\ s^{-1}, \quad t_h = 12.3\ s.$

For all cases, a = 5 cm, h = 5 cm, and Pr = 1.0. The heat-up time scale t_h [2, 3] is defined as $t_h = 2^{-1/2} Ra^{1/4} N_f^{-1}$. Since the boundary conditions are anti-symmetric about the mid-depth plane, the ensuing discussion will be restricted to the flow in the bottom half of the cylinder.

Figure 1 shows the evolution of the meridional stream function, ψ, for case N3. Here ψ is defined such that $ru = -\partial\psi/\partial z$ and $rw = \partial\psi/\partial r$. Guided by the analyses of refs. [2, 3], ψ is normalized by $Ra^{-1/4} N_f h^2$, and time is scaled by $\tau = t/t_h$. In Fig. 1, negative values of ψ imply a clockwise circulation. Because of cooling in the bottom half of the cylinder sidewall, sinking motions result in the sidewall boundary layer. This, in turn, causes the interior fluid to be sucked into the sidewall boundary layer. The fluid leaves the buoyant boundary layer near the corner, flows over the bottom endwall, and returns into the interior, thus completing the circulation. At very early times [see Fig. 1(a)], the sidewall boundary layer has not fully developed, therefore, the suction of the interior fluid is still growing with time. The boundary layer suction and the accompanying meridional circulation are very intense at early times [see Fig. 1(b); compare the magnitudes of ψ_{min} in Fig. 1]. At intermediate times and afterwards, as the heat-up process progresses, the boundary layer suction and the resulting meridional circulation weaken [see Figs. 1(c) and (d)].

As Ra varies, the qualitative patterns of the meridional flows in the interior are similar to those shown in Fig. 1. However, the thickness of the buoyant boundary layer decreases as Ra increases. This is consistent with the finding of the linear analysis of refs. [2, 3], in which the thickness of the buoyant boundary layer is scaled as $O(Ra^{-1/4} h)$.

Figure 2 depicts the plots of the isotherms, expressed in scaled dimensionless temperature $\theta = (T - T_i)/[\Delta T(z/h - 0.5)]$, along the vertical cut at $r/a = 0.5$. These plots indicate that the temperature adjustment process is characterized by the time scale t_h. The vertical temperature gradient is large at early times in the region close to the endwall. At large times, as the heat-up process is being accomplished, the temperature gradient weakens accordingly, approaching the final state value of $\Delta T/h$.

The effect of Ra on the temperature field can be seen by comparing Figs. 2(a) and (b). For a lower value of Ra, due to the increased influence of diffusion, the heat-up process proceeds

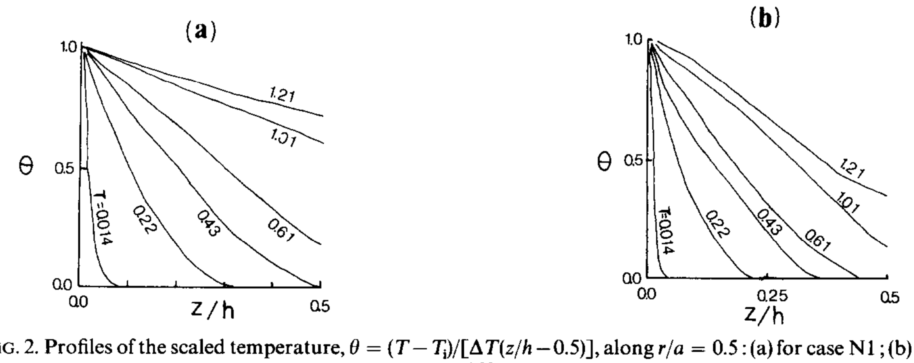

FIG. 2. Profiles of the scaled temperature, $\theta = (T - T_i)/[\Delta T(z/h - 0.5)]$, along $r/a = 0.5$: (a) for case N1; (b) for case N3.

more rapidly. The enhanced temperature adjustment process for a lower value of Ra is more pronounced at large times. This is explained by noting that the diffusive effects, as compared with the inviscid dynamic effects, become relatively more important at large times.

Figure 3 shows the vertical location of the propagating temperature front as functions of time along $r/a = 0.5$. Since the actual temperature field varies continuously, the temperature front is defined as the location at which the scaled temperature reaches some arbitrary value near zero. For convenience, one takes $\theta = 0.05$ for the temperature front. It is apparent in Fig. 3 that the propagation speed of the front is fairly constant over much of the cylinder depth. Figure 3 also suggests that the front reaches the cylinder mid-depth in approximately $0.5\,t_h$. The propagation speed of the front increases as Ra decreases. The temperature adjustment process (thus the propagation speed of the temperature front) is facilitated by larger magnitudes of the diffusive effects for lower values of Ra.

Also included in Fig. 3 as dotted lines are the plots of the locations at which $\theta = 0.5$, which lie in the region behind the front. The enhancement of the heat-up process for lower values of Ra is more pronounced in the region well behind the front than in the region near the front. Again, this is explained by noting that the relative importance of the diffusive effects increases at large times.

4. CONCLUSION

The principal mechanism for heat-up from an initial state of non-stratification is the meridional circulation driven by the buoyant boundary layer pumping. The temperature field is divided into two regions by a propagating temperature front.

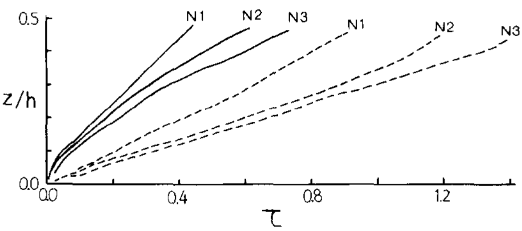

FIG. 3. Solid lines denote the locations of the temperature front ($\theta = 0.05$), and dashed lines denote the locations where $\theta = 0.5$ at $r/a = 0.5$.

The fluid ahead of the front remains non-stratified. It takes approximately $0.5\,t_h$ for the front to reach the cylinder mid-depth. The propagation speed of the front increases as Ra decreases. This is due to the increased influence of the diffusive effects.

REFERENCES

1. G. Veronis, The analogy between rotating and stratified fluids, *Ann. Rev. Fluid Mech.* **2**, 37–66 (1970).
2. T. Sakurai and T. Matsuda, A temperature adjustment process in a Boussinesq fluid via a buoyancy-induced meridional circulation, *J. Fluid Mech.* **54**, 417–421 (1972).
3. M. C. Jischke and R. T. Doty, Linearized buoyant motion in a closed container, *J. Fluid Mech.* **71**, 729–754 (1975).
4. A. Warn-Varnas, W. W. Fowlis, S. Piacsek and S. M. Lee, Numerical solutions and laser-Doppler measurements of spin-up, *J. Fluid Mech.* **85**, 609–639 (1978).